Introduction to Circuit Analysis Laboratory

Lab Experiment **12**

Mesh Analysis

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1. Method of analysis: Mesh analysis

Method of analysis is a technique used to solve complex circuit with one or more sources that cannot be solve using the tradition series, parallel or series-parallel method.

One of the method of analysis is *mesh analysis*. Mesh analysis applies Kirchhoff's Voltage Law, KVL, along with Ohm's law to solve for a circuit. The goal of mesh analysis is to find a set of simultaneous linear of equation that then can be solved to obtain the required mesh current. There are different math method to solve for the linear equation with two or more unknown variables such as elimination and/or substitution rule and Cramer's rule, which allows us to calculate variables as a quotient determinant.

1.1 Solving Systems of linear equation with two variables using elimination

For systems with two variables with different coefficient in both equations, one way to solve for the system is by elimination with multiplication. The steps for this method are:

Step 1: Decide which variable to eliminate.

Step 2: Find the Least Common Multiple (LCM) of the coefficient of both equations.

Step 3: Multiply both equations by a constant so the coefficient on both equation can be cancelled when adding them.

Step 4: Add both equation and solve the resulting equation for the other variable.

Step 5: Pick one original equation and substitute the value to find the value of the eliminated variable.

Example 1.1 – Solving linear equation with two variables using elimination

For the following system of equation with two variables I_1 and I_2, use the elimination method and solve for i_1 and i_2

$$6I_1 – 5I_2 = -27$$

$$2I_1 + 4I_2 = -26$$

Following the previous steps:

Step 1: *Decide which variable to eliminate.*

In this case, if you want to eliminate I_1, you can see that the Least Common Multiple (LCM) for both I_1 is 6. Then, if you want to eliminate I_1, the first equation must be multiplied by 1 and the second by 3. On the other hand, if you want to eliminate I_2, the LCM for both I_2 is 20. Then, to eliminate I_2, you multiply the first equation with 4 and the second with 5. Which variable should be eliminated first? It really does not matter, but I personally recommend to eliminate the variable that will result with a lower coefficient. In this case, I will eliminate I_1 first because the LCM for both equation is 6.

Step 2: *Find the Least Common Multiple (LCM) of the coefficient of both equations.*

The LCM of the coefficient of I_1 for both equations is 6.

Step 3: *Multiply both equations by a constant so the coefficient on both equation can be cancelled when adding them.*

For our example, one coefficient for I_1 must be -6 and the other +6. For it, you will multiply the first equation with -1 to make the coefficient to be -6

$$\mathbf{-1}\times (6I_1 – 5I_2 = -27) \rightarrow \mathbf{-6I_1 + 5I_2 = +27}$$

$$\mathbf{3}\times (2I_1 + 4I_2 = -26) \rightarrow \mathbf{6I_1 + 12I_2 = -78}$$

Step 4: *Add both equation and solve the resulting equation for the other variable.*

$$\mathbf{-6}I_1 + \mathbf{5}I_2 = \mathbf{+27}$$

$$\mathbf{6}I_1 + \mathbf{12}I_2 = \mathbf{-78}$$

$$\mathbf{17I_2 = -51}$$

$$\mathbf{I_2} = \frac{-51}{17} = -3$$

Step 5: *Pick one original equation and substitute the value to find the value of the eliminated variable*

$6I_1 - 5\mathbf{I_2} = -27$ ➔ since $\mathbf{I_2 = -3}$

$6I_1 - 5\mathbf{(-3)} = -27$

$6I_1 \mathbf{-15} = -27$

$6I_1 = -27 \mathbf{+ 15}$

$6I_1 = -12$

$$\mathbf{I_1} = -\frac{12}{6} = -2$$

1.2. Mesh analysis with two voltage sources

Mesh analysis applies Kirchhoff's Voltage Law, KVL, along with Ohm's law to find the simultaneous linear of equation. There are different ways to find the linear equations, this lab experiment will show you how to find those equation by general analysis using independent loops. The steps to find the linear equations are:

Step 1: Identify the number of independent loops.

Step 2: Set the direction of current flow for each independent loop and label the loop/mesh current as I_1, I_2… I_N, where *N* is the number of independent loops.

Step 3: Set the polarity of the voltage drop for the unknown voltage according to the direction of current flow set in Step 2.

Step 4: Apply KVL to each independent closed loop and write the linear equation.

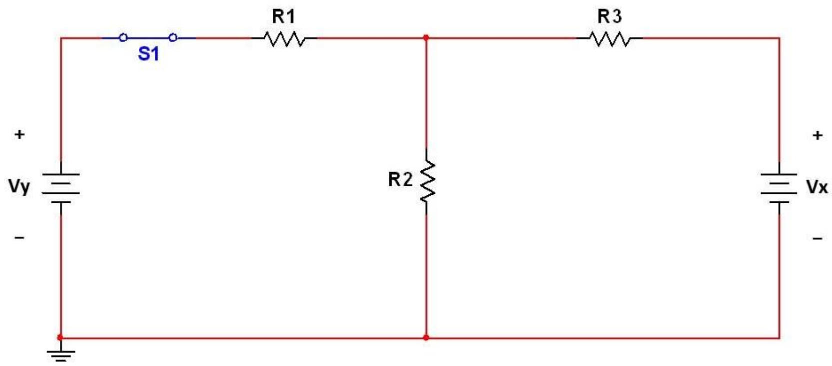
Step 5: Using the equations from Step 4, apply Ohm's law to represent the unknown voltage (voltage drop at each resistor)

Step 6: Use elimination and solve for each mesh current I_1, I_2…, I_N

Step 7: Solve for the circuit using the mesh current found in Step 6.

Example 1.2 – Mesh analysis with two voltage sources

For the following circuit 12.1, use mesh analysis to find the voltage and current through each resistor.



Circuit 12.1 – Resistive circuit with two voltage sources

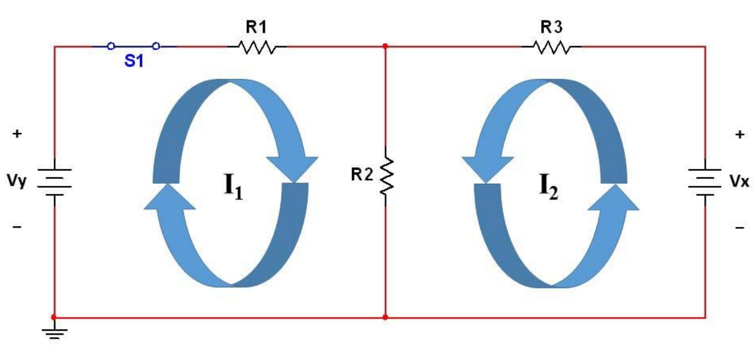
Using the steps mentioned before:

Step 1: *Identify the number of independent loops.*

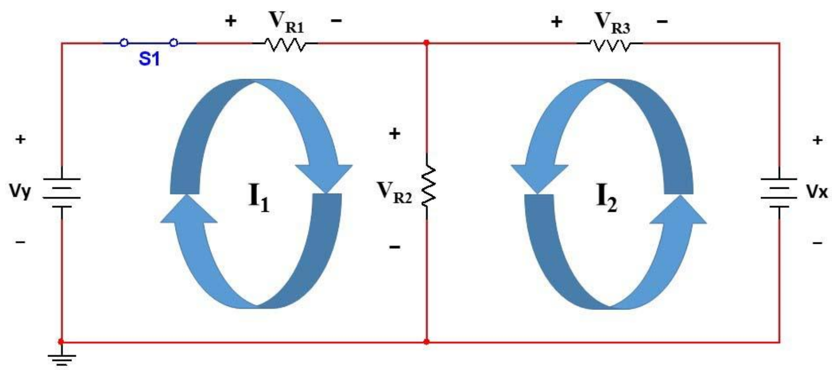
For Circuit 1.1, there are two independent loops. This means that the resulting linear equation will have two unknown.

Step 2: *Set the direction of current flow for each independent loop and label the loop/mesh current as I_1, I_2, …, I_N.*

Setting the current of flow (clockwise ⤻ or counterclockwise ⤺) is ready up to the student, at the end, when you solve for the linear equations, the magnitude of the mesh current will be the same. But you need to keep in mind that if the resulting mesh current is positive, it means that the mesh current was set to the conventional flow of current. On the other hand, if the mesh current is negative, it means that the mesh current was set to the electrons flow of current. Usually, I use the polarity of the voltage sources to set the direction of the current flow. For example, for Circuit 1.1, I will set the mesh current for Vy clockwise because the current will flow from negative to positive. Likewise, the mesh current for Vx counterclockwise.



Step 3: *Set the polarity of the voltage drop for the unknown voltage according to the direction of current flow set in Step 2.*



Step 4: *Apply KVL to each independent closed loop and write the linear equation.*

<u>KVL for mesh I_1</u>

$Vy - V_{R1} - V_{R2} = 0V$

$Vy = V_{R1} + V_{R2}$

<u>KVL for mesh I_2</u>

$Vx - V_{R3} - V_{R2} = 0V$

$Vx = V_{R3} + V_{R2}$

Step 5: *From the equation in Step 4, use Ohm's law to represent the unknown voltage (voltage drop at each resistor)*

<u>KVL for mesh I_1</u>

$Vy = V_{R1} + V_{R2}$

The Ohm's law equation for V_{R1} is:

$V_{R1} = I_1 \times R_1$

To find the Ohm's law equation for V_{R2},

Since there are two mesh currents going through R_2, which are I_1 and I_2, and both currents flow down through R_2, then the total current through R_2 is the sum of I_1 and I_2. On the other hand, if the mesh currents through R_1 flow in different direction, then the total current through R_2 shall be their difference.

$$V_{R2} = (I_1 + I_2) \times R_2 = I_1 \times R_2 + I_2 \times R_2$$

Another observation on this step is the order of I_1 and I_2. I_1 goes before I_2 because we are writing the KVL equation for mesh I_1. This procedure is important if we are subtracting the mesh currents.

Replacing the Ohm's law equation into the KVL equation:

$Vy = I_1 \times R_1 + I_1 \times R_2 + I_2 \times R_2$

$Vy = I_1 \times R_1 + I_1 \times R_2 + I_2 \times R_2$

$Vy = I_1(R_1 + R_2) + I_2 \times R_2$ ➔ Linear equation for mesh I_1

<u>KVL for mesh I_2</u>

$Vx = V_{R3} + V_{R2}$

The Ohm's law equation for V_{R3} is:

$V_{R3} = I_2 \times R_3$

To find the Ohm's law equation for V_{R2},

$V_{R2} = (I_2 + I_1) \times R_2 = I_2 \times R_2 + I_1 \times R_2$

I_2 goes before i_1 because we are writing the KVL equation for mesh I_2.

Replacing the Ohm's law equation into the KVL equation:

$Vx = I_2 \times R_3 + I_2 \times R_2 + I_1 \times R_2$

$Vy = I_1 \times R_2 + I_2(R_3 + R_2)$ ➔ Linear equation for mesh I_2

Step 6: *Use elimination and solve for each mesh current I_1 and I_2*

Step 7: *Solve for the circuit using the mesh current found in Step 6.*

Once you have the mesh current, according to Circuit 12.1

$I_{R1} = I_1$	$V_{R1} = I_{R1} \times R_1$
$I_{R2} = I_1 + I_2$	$V_{R2} = I_{R2} \times R_2$
$I_{R3} = I_2$	$V_{R3} = I_{R3} \times R_3$

<u>Appendix</u>

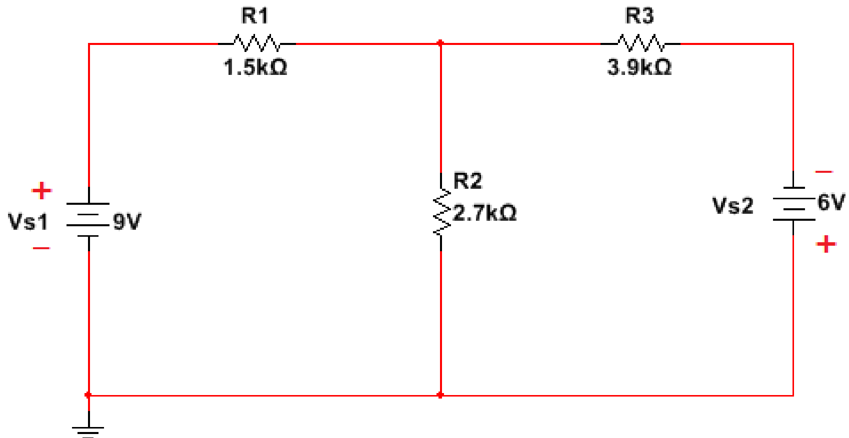
$$\%\ of\ difference = \left(\frac{Measured\ Value - Calculated\ Value}{Calculated\ Value}\right) \times 100\%$$

Formula 12.1 - % of difference formula

Lab Experiment Procedure

Part 1 – Resistivity Circuit with Two Voltage Sources

1. Obtain 1.5 kΩ, 2.7 kΩ , and 3.9 kΩ resistors from your component kit, measure their resistance, and record the measured values in Table 12.1
2. Build Circuit 12.2 in a protoboard. For this step, it is very important to remember which power nodes are used for Vs1 and Vs2. For example, you can identify the top + and – node as for Vs1 and the bottom + and – node as for Vs2.



Circuit 12.2 – Resistive Circuit with Two Voltage Sources

3. Since Circuit 12.1 is using two power supplies with different voltages, before connecting the power supplies to the circuit, the two power supplies have to be connected in a way that they will have one common ground.
4. Prepare the Circuit 12.2 and the DMM to measure current.
5. Set one power supply to 9 V and the other to 6 V and connect them to the circuit in the protoboard.
6. Measure the current through R_1, R_2, and R_3 and record the values in Table 12.2

	Measured Value (Include Unit)	Calculated value (Include Unit)	% of Difference
Current through R1 (1.5 kΩ), I_{R1}			
Current through R2 (2.7 kΩ), I_{R2}			
Current through R3 (3.9 kΩ), I_{R3}			

Table 12.2 Current Measurement through each resistor of Circuit 12.2

7. Turn OFF the power source.
8. Prepare the multimeter and the circuit to measure voltage.
9. Measure the voltage through R_1, R_2, and R_3 and record the values in Table 12.3

	Measured Value (Include Unit)	**Calculated value (Include Unit)**	**% of Difference**
Voltage through R1 (1.5 kΩ), V_{R1}			
Voltage through R2 (2.7 kΩ), V_{R2}			
Voltage through R3 (3.9 kΩ), V_{R3}			

Table 12.3 Voltage Measurement through each resistor of Circuit 12.2

10. Turn OFF the power source
11. Use mesh analysis to calculate the current and voltage through each resistor. Record the calculation in Table 12.2 and 12.3 respectively.

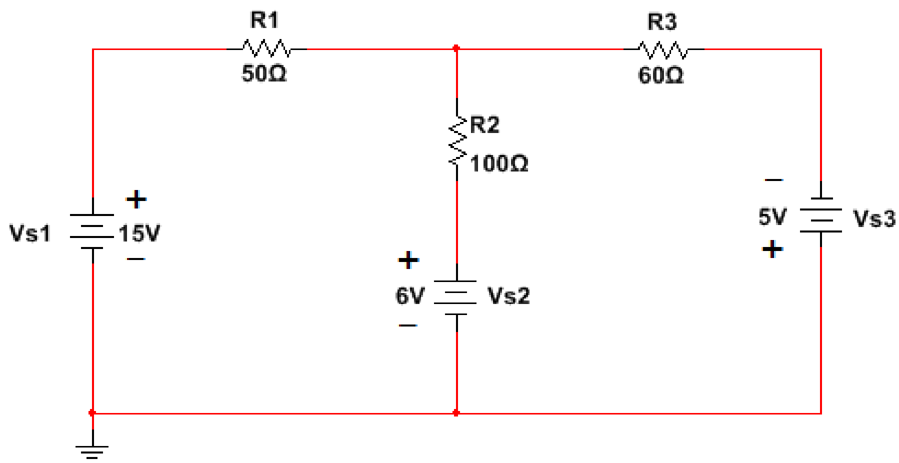
Show mesh analysis calculations here

12. Find the percentage of differences between the measured and calculated value and record the answer in Table 12.2 and 12.3 respectively.
13. Ask lab instructor to check the calculations and tables. Once the tables are checked, disassembled the circuit, and organize your components in your components kit.

Part 2 – Exercises: Analyzing circuits using mesh analysis

For this part of the lab, you will practice how to analyze different type of circuit using mesh analysis.

Exercise 12.1 Given Circuit 12.3, use mesh analysis to:



Circuit 12.3 – Resisttve Circuit with Two Voltage Sources

a. Find the linear equation (from Kirchhoff's Voltage Law and Ohm's law) for mesh current I_1 and I_2

Equation Mesh I_1________________________ Equation Mesh I_2_______________________

Show Calculations Here

b. Using the mesh equations, apply elimination method or Cramer's law to find the mesh current I_1 and I_2

I_1 = ______________________ , I_2 = _____________________

Show Calculations Here

c. Using the mesh current i_1 and i_2, find the current through each resistor:

I_{R1} = _____________________ I_{R2} = _____________________ I_{R3} = _____________________

Show calculations here

d. Find the voltage across each resistor using Ohm's law:

V_{R1} = ____________________ V_{R2} = ____________________ V_{R3} = ____________________

Show calculations here

Questions

1. For Circuit 12.2, if a student by mistake reversed/flipped the polarity of Vs2 when building the circuit in a protoboard, how this mistake would affect his measurements? Explain and/or justify your answer.
2. Analyzing a circuit using mesh analysis, a student is measuring the current through the resistors R_1, R_2, and R_3. The student measured that the current through R_1 is the mesh current I_1 and the current through R_2 is the mesh current I_2. What experimental procedure should a student apply to find if the mesh currents I_1 and I_2 are flowing clockwise or counter-clockwise? Explain and/or justify your answer.

Answer

Student's Signature: _____________ Lab Instructor's Signature ______________Date: _______

-------------------------------------- LAB EXPERIMENT ENDS HERE -------------------------------------