Chapter 3

Fundamental Laws: Ohm's Law, Kirchhoffs's Law, and Power's Law

As we already learned in Chapter 2 about circuit terminology and basic circuit concepts such as current, voltage, and power in an electric circuit. In chapter 3 we will learn how these basic concepts are applied in an electric circuit by means of fundamental laws that govern them. These laws are known as Ohm's law, Kirchhoff's laws, and power's law.

But the end of the chapter, students should be able to:

- Understand and apply Ohm's law to an unknown element value, current, resistance, or voltage.
- Comprehend the application of Kirchhoff's law and demonstrate ability to solve an unknown current and voltage within a network.
- Apprehend the power analysis in a network with respect to current, resistance, and voltage.

3.1. Ohm's Law

(Material about Ohm's law was retrieved on All About Circuits)

The first, and perhaps most important, the relationship between current, voltage, and resistance is called Ohm's Law, discovered by Georg Simon Ohm and published in his 1827 paper, The Galvanic Circuit Investigated Mathematically.

3.1.1. The Ohm's Law Equation

Ohm's principal discovery was that the amount of electric current through a metal conductor in a circuit is directly proportional to the voltage impressed across it, for any given temperature. Ohm expressed his discovery in the form of a simple equation, describing how voltage, current, and resistance interrelate: $E = I\,R$

In this algebraic expression, voltage (E) is equal to current (I) multiplied by resistance (R). Using algebra techniques, we can manipulate this equation into two variations, solving for I and for R, respectively:

$$I = \frac{E}{R} \qquad R = \frac{E}{I}$$

Analyzing Simple Circuits with Ohm's Law

Let's see how these equations might work to help us analyze simple circuits:

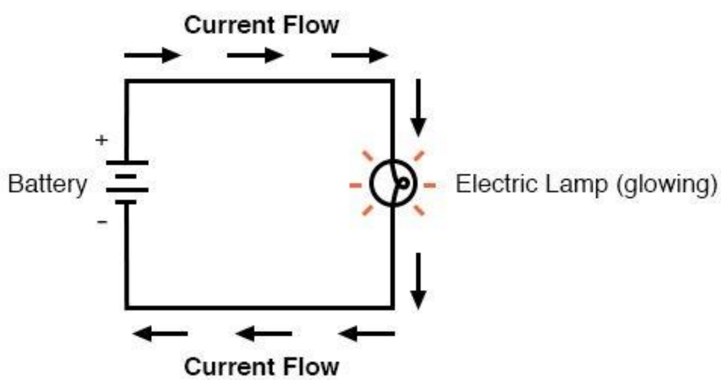


Figure 3.1 –Closed Circuit

In the above circuit, there is only one source of voltage (the battery, on the left) and only one source of resistance to current (the lamp, on the right). This makes it very easy to apply Ohm's Law. If we know the values of any two of the three quantities (voltage, current, and resistance) in this circuit, we can use Ohm's Law to determine the third.

In this first example, we will calculate the amount of current (I) in a circuit, given values of voltage (E) and resistance (R):

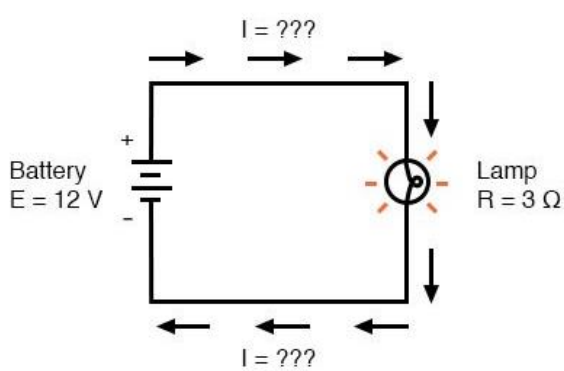


Figure 3.2 – Finding Current Flow in a Closed Circuit

What is the amount of current (I) in this circuit?

$$I = \frac{E}{R} = \frac{12\ V}{3\ \Omega} = 4\ A$$

In this second example, we will calculate the amount of resistance (R) in a circuit, given values of voltage (E) and current (I):

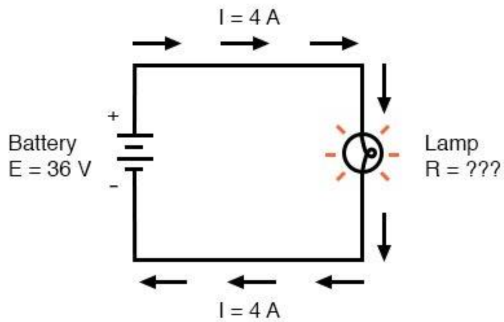


Figure 3.3 – Finding Resistance in a Closed Circuit

What is the amount of resistance (R) offered by the lamp?

$$R = \frac{E}{I} = \frac{36\ V}{4\ A} = 9\ \Omega$$

In the last example, we will calculate the amount of voltage supplied by a battery, given values of current (I) and resistance (R):

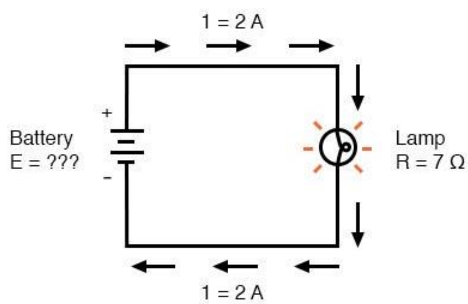


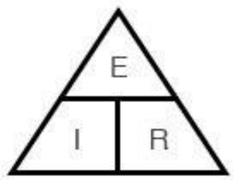
Figure 3.4 – Voltage in a Closed Circuit

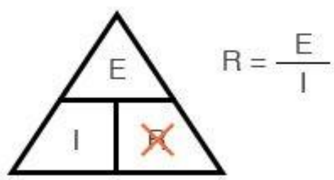
What is the amount of voltage provided by the battery?

$$E = IR = (2\ A)(7\ \Omega) = 14\ V$$

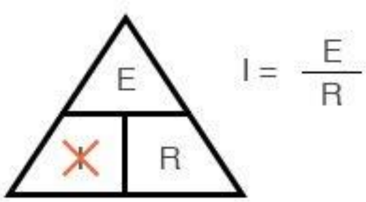
3.1.2. Ohm’s Law Triangle Technique

Ohm’s Law is a very simple and useful tool for analyzing electric circuits. It is used so often in the study of electricity and electronics that it needs to be committed to memory by the serious student. For those who are not yet comfortable with algebra, there’s a trick to remembering how to solve for anyone quantity, given the other two. First, arrange the letters E, I, and R in a triangle like this:

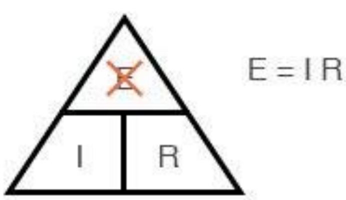




If you know E and I, and wish to determine R, just eliminate R from the picture and see what’s left:



If you know E and R, and wish to determine I, eliminate I and see what’s left:



Lastly, if you know I and R, and wish to determine E, eliminate E and see what’s left:

Eventually, you’ll have to be familiar with algebra to seriously study electricity and electronics, but this tip can make your first calculations a little easier to remember. If you are comfortable with algebra, all you need to do is commit E=IR to memory and derive the other two formulae from that when you need them!

(Ohm's Law - How Voltage, Current, and Resistance Relate, 2020)

3.2. Kirchhoff's Law

Kirchhoff's Circuit Laws is the second fundamental laws in circuit analysis. The idea of Kirchhoff's laws is commonly known as the **Conservation of Energy**, which states that the **energy** is neither created nor destroyed. In circuitry, Kirchhoff's law is implicit that the directed sum of the **electrical** potential differences (voltages) around any closed circuit is equal to zero, or that the sum of all currents flowing in a node is equal to zero.

(The following material about Kirchhoff's laws, except some examples, was retrieved on https://www.khanacademy.org/)

Kirchhoff's Laws describe current in a node and voltage around a loop. These two laws are the foundation of advanced circuit analysis. Kirchhoff's Laws for current and voltage lie at the heart of circuit analysis. With these two laws, plus Ohm's law, and the equations for individual component (resistor, capacitor, inductor), we have the basic tool set we need to start analyzing circuits.

3.2.1. Kirchhoff's loop rule, Kirchhoff's Voltage Law (KVL)

Kirchhoff's loop rule states that the sum of all the electric potential differences, voltage, around a loop is zero. It is also sometimes called Kirchhoff's Voltage Law, KVL, or Kirchhoff's second law. This means that the energy supplied by the battery is used up by all the other components in a loop, since energy can't enter or leave a closed circuit. The rule is an application of the conservation of energy in terms of electric potential difference, ΔV[1]

Mathematically, this can be written as:

$$\boldsymbol{V_{\circlearrowleft} = \Sigma \Delta V = 0\ V}$$

Formula 3.1 – Kirchhoff's Voltage Law, KVL

Formula 3.1 translates as the sum of all voltages in a closed loop is equal to zero.

[1] Δ is a Greek symbol, uppercase delta, and it is used in math to represent changes. Σ is a also a Greek symbol, sigma, which represents summation.

How to determine the electric potential difference across a circuit component

For example, we can use Kirchhoff's loop rule to find the unknown electric potential difference across circuit element 1, Y_1 (Figure 3.5).

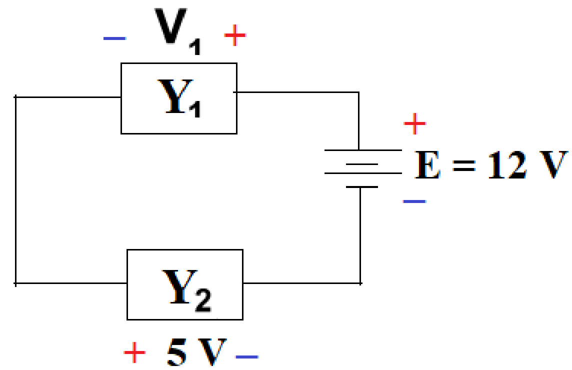


Figure 3.5: A circuit with two circuit elements, Y_1 and Y_2, and one voltage source, E

Step 1: Pick a starting point, let's say the voltage source, and pick a direction of current flow, clockwise or counterclockwise. For this example, let's pick the voltage source as the starting point and set counterclockwise as the direction of the current flow going.

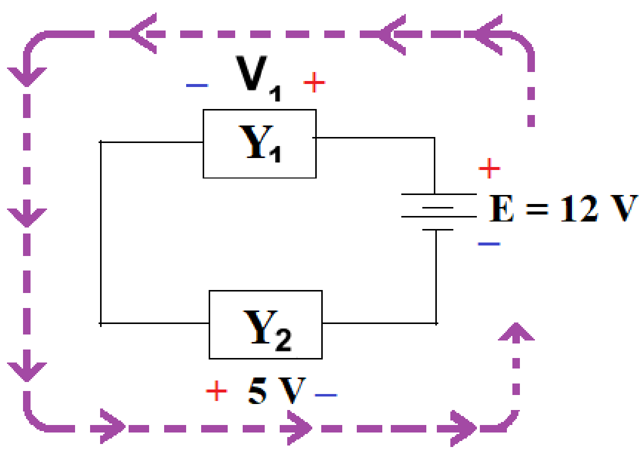


Figure 3.5A: Circuit in Figure 3.5 with current flowing counterclockwise

Step 2: Now, let's use KVL and write all the voltages within the closed circuit in Figure 3.5A starting with the voltage source, E.

$$V_{\circlearrowleft} = \Sigma \Delta V = 0\,V$$

$$+12\,V - V_1 - 5\,V = 0\,V$$

Step 3: Once the KVL formula is set, we solve for the unknown voltage, V_1

$$+12\,V - \cancel{V_1} - 5\,V = 0\,V$$

$$+\cancel{V_1} \qquad = +V_1$$

$$+12\,V - 5\,V = V_1$$

$$+7\,V = V_1$$

What happens if I go the wrong way?

There is no "wrong" way to go around a circuit. Just make sure to use your positive and negative signs appropriately!

If we go opposite the direction of conventional current, then the electric potential increases across the circuit elements, so voltage across Y_1 and Y_2 is positive,

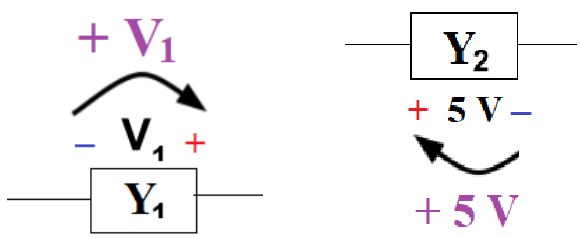


Figure 3.5B: Voltage drop in circuit elements Y1 and Y2 with current flowing clockwise

and electric potential decreases across batteries.

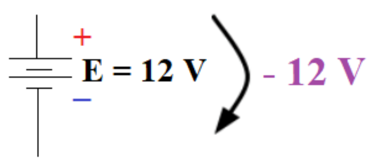


Figure 3.5C: Voltage drop in voltage source with current flowing clockwise

Now, the KVL equation with the current flowing clockwise will be:

$$V_{\circlearrowleft} = \Sigma\,\Delta\,V = 0\,V$$

$$-12\,V + V_1 + 5\,V = 0\,V$$

And solving for the unknown voltage V_1 will be also 7 V.

3.2.2. Kirchhoff's junction rule or Kirchhoff's Current Law, KCL

Kirchhoff's junction rule says that the total current flowing into a junction, node, equals the total current flowing out of the same junction. This is a statement of conservation of charge. It is also sometimes called Kirchhoff's first law, Kirchhoff's current law, the junction rule, or the node rule. Mathematically, we can write it as:

$$I_{into(node\ A)} = I_{out(node\ A)}$$

Formula 3.2 – Kirchhoff's Current Law, KCL

Junctions can't store current, and current can't just disappear into thin air because charge is conserved. Therefore, the total amount of current flowing through the circuit must be constant.

For example, we can use the KCL equation to find the balance of all currents to a node in Figure 3.6.

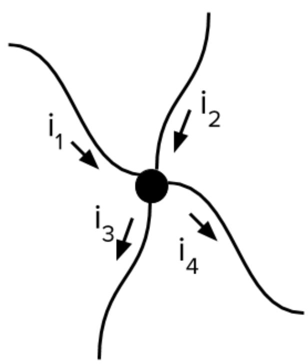


Figure 3.6: Current distribution in a junction or node

$$I_{into(node\ A)} = I_{out(node\ A)}$$

$$i_1 + i_2 = i_3 + i_4$$

Now, in Figure 3.7, the current into the node equals the current out of the node.

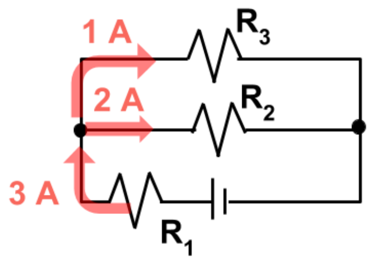


Figure 3.7: The current into the node equals the current out of the node

The current into the node is 3 A. There are two branches out of the node. The current across resistor R_2 is 2 A, and the current across resistor R_3 is 1 A, so we can write the KCL equation as:

$$I_{into(node\ A)} = I_{out(node\ A)}$$

$$3\ A = 2\ A + 1\ A$$

$$3\ A = 3\ A$$ ✓

(Khan Academy, 2020)

Example) Use KCL to find the unknown currents, I_X and I_Y, in Figure 3.8

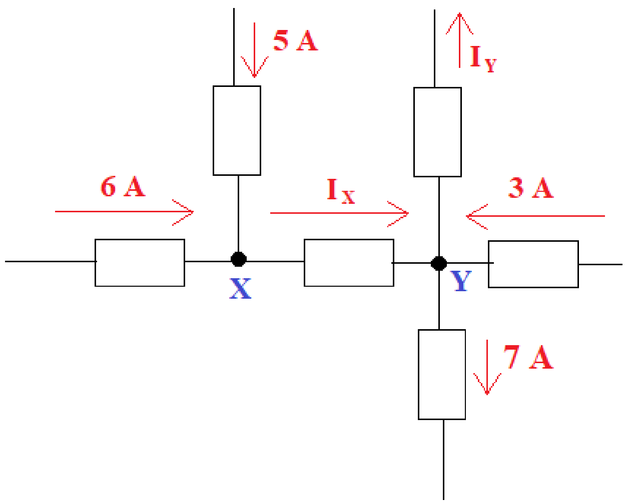


Figure 3.8: Current distribution through node X and Y in a network

Since we have two nodes, let's start applying KCL to one node at the time. Since node X has only one unknown current, I_X, we can apply KCL to node X first:

$$I_{into(node\ X)} = I_{out(node\ X)}$$
$$I_X + 5\ A = I_Y + 7A$$
$$11\ A + 5\ A = I_Y + 7A$$
$$11\ A + 5A - 7A = I_Y$$
$$9\ A = I_Y$$

Once we have found current I_X, now we can apply KCL to node Y to find I_Y:

$$I_{into(node\ Y)} = I_{out(node\ Y)}$$
$$I_Y + 3\ A = I_X$$
$$11\ A = I_X$$

3.3. Power's Law

In electric circuit, power is the rate at which energy is transferred to or from a part of an electric circuit or element, measured in watts (W). For example, a voltage source can deliver energy to elements in the circuit, or a resistor can dissipate energy as heat. Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it:

$$P = V * I$$

Formula 3.3 – Power Formula

Using the power formula, in case resistance is given, we can use Ohm's law to substitute the voltage or current to find power.

Let's use the power formula and substitute V:

$P = V * I$ $\longrightarrow$ Substitute $V = I * R\ (Ohm's\ law)$

$P = I * R * I$

$P = I^2 * R$

Now, let's use the power formula and substitute I:

$P = V * I$ $\longrightarrow$ Substitute $I = \frac{V}{R}(Ohm's\ law)$

$P = V * \frac{V}{R}$

$P = \frac{V^2}{R}$

Example) An energy source forces a constant current of 2.5 A to flow through a light bulb of 10Ω, calculate the power dissipates through the light bulb.

Answer:

$P = I^2 * R$

Substitute the given values I = 2.5 A and R = 10 Ω

$P = (2.5\ A)^2 * (10\ \Omega)$

P = 62.5 W

Example) A 1.5 kΩ resistor dissipates 600 mW of power, what would be the current through it?

$P = I^2 * R$

Substitute the given values R = 1.5 kΩ = 1.5×10^3 Ω and P = 600 mW = 600×10^{-3} W

$600 \times 10^{-3}\ \mathrm{W} = I^2 * (1.5 \times 10^3\ \Omega)$

$\frac{600\times10^{-3}\ \mathrm{W}}{1.5\times10^3\ \Omega} = I^2$ ➔ $\sqrt{\frac{600\times10^{-3}\ \mathrm{W}}{1.5\times10^3\ \Omega}} = \sqrt{I^2}$

$\sqrt{400 \times 10^{-3-(3)}\ \frac{\mathrm{W}}{\Omega}} = I$ ➔ $\sqrt{400 \times 10^{-6}\ \frac{\mathrm{W}}{\Omega}} = I$

$20 \times 10^{-3} A = I$

$I = 20\ mA$

References

Khan Academy. (2020). Retrieved from Kirchhoff's Junction and Loop Law: https://www.khanacademy.org/science/physics/ap-physics-1/ap-circuits-topic#kirchhoffs-junction-rule

Ohm's Law - How Voltage, Current, and Resistance Relate. (2020). Retrieved from All About Circuits: https://www.allaboutcircuits.com/textbook/direct-current/chpt-2/voltage-current-resistance-relate/