Chapter 1

Technical Mathematics

Chapter 1 covers the technical mathematics that an engineering technology student needs to know in order to solve and complete the analysis of a circuit under dc.

1.1 Powers of Ten

It should be apparent from the relative magnitude of the various units of measurement that very large and very small numbers are frequently encountered in the sciences. To ease the difficulty of mathematical operations with numbers of such varying size, powers of ten are usually employed. This notation takes full advantage of the mathematical properties of powers of ten. The notation used to represent numbers that are integer powers of ten is as follows:

$$\begin{array}{rclcrcrcl}
1 & = & 10^{0} & \qquad & \frac{1}{10} & = & 0.1 & = & 10^{-1} \\
10 & = & 10^{1} & & \frac{1}{100} & = & 0.01 & = & 10^{-2} \\
100 & = & 10^{2} & & \frac{1}{1,000} & = & 0.001 & = & 10^{-3} \\
1000 & = & 10^{3} & & \frac{1}{10,000} & = & 0.0001 & = & 10^{-4}
\end{array}$$

where, an expression 10^4 is called a **power**, read “ten to the fourth power.” The **exponent** 4 represents the number of times the **base** 10 is used as a factor as shown below.

$$\text{base} \rightarrow \underbrace{10^{\overset{\text{exponent}}{4}}}_{\text{Power}} = \underbrace{10 \cdot 10 \cdot 10 \cdot 10}_{\text{4 factors of 10}}$$

A quick method of determining the proper power of ten is to place a caret mark to the right of the numeral 1 wherever it may occur; then count from this point to the number of places to the right or left before arriving at the decimal point. Moving to the right indicates a positive power of ten, whereas moving to the left indicates a negative power. For example,

$$100{,}000 = 1\,0\,0\,,0\,0\,0\,. = 10^{+5}, \qquad 0.0001 = 0\,.\,0\,0\,0\,1 = 10^{-4}$$

Properties of Powers of Ten

1) Zero exponent: $10^0 = 1$

2) Negative exponent: $\frac{1}{10^n} = 10^{-n}$, $\frac{1}{10^{-n}} = 10^n$

Example 1.1

a. $\frac{1}{10^4} = 10^{-4}$

b. $\frac{1}{10^{-5}} = 10^5$

3) Product of powers of ten: $(10^m)(10^n) = 10^{(m+n)}$

Example 1.2

a. $(1000)(10{,}000) = (10^3)(10^4) = 10^{(3+4)} = 10^7$

b. $(0.000001)(100) = (10^{-6})(10^2) = 10^{(-6+2)} = 10^{-4}$

4) Quotient of powers of ten: $\frac{10^m}{10^n} = 10^{(m-n)}$

Example 1.3

a. $\frac{100{,}000}{1000} = \frac{10^5}{10^3} = 10^{(5-3)} = 10^2$

b. $\frac{0.0001}{100} = \frac{10^{-4}}{10^2} = 10^{(-4-2)} = 10^{-6}$

5) Power of a power of ten: $(10^m)^n = 10^{mn}$

Example 1.4

a. $(1000)^4 = (10^3)^4 = 10^{3\times 4} = 10^{12}$

b. $(0.00001)^3 = (10^{-5})^3 = 10^{-15}$

1.2. Scientific and Engineering notation

In electronics, technicians very often have to deal with measurable values that might be very large or very small numbers. For example, the distance from the Earth to the sun, which is 92960000 miles, or the thickness of the aluminum foil, which is 0.000963 inches. These numbers are impractical to write out because of the length, the amount of space required, and the difficulty to reading them. Due to it, scientists have developed a shorter method to write very large or very small numbers. Those methods are known as scientific notation and engineering notation.

Scientific Notation

Scientific notation is based on powers of 10. It is a method to represent very large or very small number by representing the number with a coefficient, named Mantissa, greater or equal to 1 and less than 10, times powers of 10. For example, the distance from the Earth to the sun written in scientific notation is $\boldsymbol{9.296\times10^{7}}$ miles. In this case, the number ***9.296*** is the ***mantissa*** which must be a number greater or equal to ***1*** and less than ***10***. The second part must be powers of 10.

Scientific notation: $c \times 10^{n}$ where $1 \leq$ **mantissa** (c) < 10 and the **exponent** n is an integer.

How to write a number in scientific notation?

To write the distance from the Earth to the sun which is 92960000 miles in scientific notation:

Step 1: Identify the number where the decimal point should be placed, so the mantissa will be greater or equal to 1 and less than 10. In this case, the decimal point must be placed in between 9 and 2 to make the mantissa to ***9.296***.

92960000

Step 2: Check how many decimal places you must move from the lowest digit of the given number so the mantissa will become ***9.296***. In this case, the decimal point must move 7 decimal places.

92960000

Step 3: Now, pay attention if the decimal point must be shifted to the left or to the right.

> **Always remember:**
> - If the decimal point is shifted to the ***left***, the base exponent ***increases***. (*positive* exponents)
> - If the decimal point is shifted to the ***right***, the base exponent ***decreases***. (*negative* exponents)

In this case, the decimal point is shifted to the left by 7 places, meaning that the base exponent is increased by 7.

92960000

+7 6 5 4 3 2 1

Step 4: Write the number in scientific notation

$$\mathbf{9.296 \times 10^{7}}$$

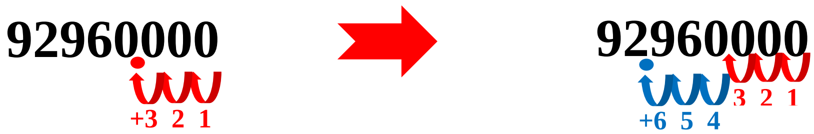
Engineering Notation

Scientific Notation is a notation widely used in science field to display very large or very small numbers. But a common method used in the field of engineering or engineering technology is the Engineering Notation. In Engineering Notation, numbers are expressed with power of ten with a *base exponent that is divisible by 3* and a *mantissa greater or equal to 1 and less than 1000*. For example, to write the distance from the Earth to the sun in engineering notation will be: 92.96×10^{6} miles.

> Engineering notation: $\boldsymbol{m} \times \mathbf{10}^{\boldsymbol{n}}$
>
> where $1 \leq$ **mantissa** (m) $< 1{,}000$ and the **exponent** n is restricted to multiples of 3.

How to write a number in engineering notation?

To write the distance from the Earth to the sun which is 92960000 miles in engineering notation:
Step 1: Shift the decimal point three places and stop to check if the mantissa is greater or equal to 1 and less than 1000. If the mantissa is in between this range, then stop shifting the decimal point. If the mantissa is not between the ranges, shift the decimal point three more places, stop and check the mantissa again. Continue to do so until the mantissa is between the ranges.



If we shifted the decimal point three times, the mantissa becomes 92960.000. Since 92960 is not less than 1000, then we need to shift the decimal point three more places.

If we shifted a total of 6 decimal places, the mantissa becomes 92.96. Since 92.96 is less than 1000 but greater or equal to 1, then we stop the shifting, and 92.96 is the mantissa in engineering notation.
Note: There is no need to write the zeros of the right side of the mantissa because there are not significant.

Also, always pay attention if the decimal point must be moved to the left or to the right. If the decimal point is shifted to the left, the base exponent increases. If the decimal point is shifted to the right, the base exponent decreases. In this case, the decimal place is shifted 6 places to the left, then the base exponent is +6.

Step 2: Write the number in engineering notation

$$92.96 \times 10^{6}$$

1.3. Prefixes

Prefixes are alternative way to write the powers of ten. It is very useful in engineering notation because it has a specific name to each power of ten which make them easy to write and read. Some of the prefixes for engineering notation are listed in Table 1.1

Prefixes			
Name	**Symbol**	**Power of ten**	**Decimal value**
exa	E	10^{18}	1,000,000,000,000,000,000
peta	P	10^{15}	1,000,000,000,000,000
tera	T	10^{12}	1,000,000,000,000
giga	G	10^{9}	1,000,000,000
mega	M	10^{6}	1,000,000
kilo	k	10^{3}	1,000
-	-	10^{0}	1
milli	m	10^{-3}	0.001
micro	µ	10^{-6}	0.000001
nano	n	10^{-9}	0.000000001
pico	p	10^{-12}	0.000000000001
femto	f	10^{-15}	0.000000000000001
atto	a	10^{-18}	0.000000000000000001

Table 1.1 *Most common powers of ten used in electrical and electronic work*

For example, the distance from the Earth to the sun, which is 92960000 miles, written in engineering notation using the respective prefix symbol will be:

$$\overbrace{92.96 \times 10^{6}\ \text{miles}}^{\text{Replace the unit miles with its abbreviation "mi"}} = 92.96\ \underbrace{\text{Mmi}}_{\text{Replace } 10^{6} \text{ with prefix symbol "M"}}$$

Table 1.2 displays each decimal quantity in engineering notation with its respective prefixes.

Use of prefixes in power of ten		
Quantity in Decimal notation	**Quantity in Engineering notation**	**Quantity in Prefix notation**
120,000,000,000 hertz	120×10^{9} Hz	120 GHz
30,000,000 bytes	30×10^{6} b	30 Mb
14,500 ohms	$14.5 \times 10^{3}\ \Omega$	14.5 kΩ
9 volts	9×10^{0} V	9 V
0.092 amperes	92×10^{-3} A	92 mA
0.000005 henrys	5×10^{-6} H	5 µH
0.0000000385 seconds	38.5×10^{-9} s	38.5 ns
0.0000000000012 farads	1.2×10^{-12} F	1.2 pF

***Table 1.2** Typical electrical quantities in decimal, engineering and prefix notation*

Example 1.1. Convert 23000 W in engineering notation using prefixes

$$\mathbf{23000.\ W = 23.000 \times 10^{3}\ W = 23.0\ kW}$$

Example 1.2. Convert 0.0000215 s in engineering notation using prefixes

$$\mathbf{0.000021.5\ s = 21.5 \times 10^{-6}\ s = 21.5\ \mu s}$$

Converting between prefixes

There are different methods to convert numbers of the same unit to a different prefix. One of the method is by using the power of ten. For example, if the number 0.03205 ms (milli-seconds) is converted to ns (nano – seconds), the steps to follow are:

Step 1: Convert each prefix by its corresponding power of ten.

0.03205 ms → ns

0.03205×10^{-3} s → 10^{-9} s

Step 2: Indicate the distance from one exponent to the other exponent.

10^{-3} → 10^{-9}

From -3 to -9 there are 6 decimal places.

Step 3: Determine if the distance of decimal places should be shifted to the right or to the left.

Always remember:

- If the exponent is converting ***from a larger to a lower exponent***, the decimal point of the number ***must be shifted to the right***.
- Otherwise, if the exponent is converting ***from a lower to a larger exponent***, the decimal point of the number ***must be shifted to the left***.

From Step 2, the exponent is converting from the larger exponent to a lower exponent, therefore, the decimal point of the number must **be shifted six places to the right**.

0.03205 ms → ns

10^{-3} → 10^{-9}

Note: Any empty spaces after or before the decimal point is filled with zero

0.032050

Step 4: Write the answer using prefixes

32050×10^{-9} s → 32050 ns

1.4. Order of Operations

Order of operation in math, including the use in a calculator, and computer programming is a set of rules where indicates which procedures to perform first in order to solve for a mathematical expression. Indeed, the order of operation in math is **P**arentheses, **E**xponents, **M**ultiplication and **D**ivision, and **A**ddition and **S**ubtraction or simply **PEMDAS**. The operations of multiplication and division have the same level of priority. To decide when to multiply or divide, always perform the one which appears first from left to right. In the same manner, addition and subtraction are co-equal in terms of importance. Perform the operation that comes first as you work it out from left to right.

For example, evaluate $-9+3\times(2-8)\div 6+2$ using the order of operations

Parenthesis	$-9+3\times(-6)\div 6+2$
Exponent	None
Multiplication	$-9-18\div 6+2$
Division	$-9-3+2$
Addition	$-9-1$
Subtraction	-10

Then $-9+3\times(2-8)\div 6+2=-10$

Try to confirm the answer in a calculator by entering the whole mathematical expression, $-9+3\times(2-8)\div 6+2$ in the calculator.

When you have an expression where the division comes before multiplication, then you perform the division operation first and then multiplication.

For example, evaluate $(3+8)+112\div 7\times 2^3$

Parenthesis	$(11)+112\div 7\times 2^3$
Exponent	$11+112\div 7\times 8$
Division	$11+16\times 8$
Multiplication	$11+128$
Addition	139
Subtraction	None

Then $(3+8)+112\div 7\times 2^3=139$

Try to confirm the answer in a calculator by entering the whole mathematical expression, $(3+8)+112\div 7\times 2^3$, did you have the same answer?

1.5. Equation with unknown variables

Solving equations that contain one unknown variable is basically to make the unknown variable to be equal to a value or equation. To do so, the rule of operation to the other side of the equal side is applied.

Example 1.3 – Solving equations with an unknown variable
Given the equation $3x - 5 = 16$, solve for the unknown value $\boldsymbol{x}$

Solution:
Solving for $\boldsymbol{x}$ means to find what $\boldsymbol{x}$ is equal to, to do so:

Add **5** to both sides of the equation ➔ $3x - 5 + 5 = 16 + 5$ ➔ $3x = 21$

Divide both sides of the equation by 3 ➔ $\frac{3x}{3} = \frac{21}{3}$ ➔ $\boxed{x = 7}$

Example 1.4 – Solving equations with the variable on both sides
If there are variables in both sides of the equation, first move all like variables to one side and the numbers to the other side. Try to collect the variables on the side of the equation where the coefficient will be positive.

Given $-5z - 26 = 12z + 8$, solve for z

Solution:
Check which side has the variable with the greater coefficient. In this case, the right side has 12z and the left side has -5z. Since 12z is greater than -5z, then we move -5z to the right side by adding 5z to both sides.

$-5z - 26 + 5z = 12z + 8 + 5z$ ➔ $-26 = 17z + 8$

Now, collect all numbers to the left side by subtracting 8 on both sides.

$-26 - 8 = 17z + 8 - 8$ ➔ $-34 = 17z$

To solve for $\boldsymbol{z}$ we need to divide both sides by 17

$\frac{-34}{17} = \frac{17z}{17}$ ➔ $\boxed{-2 = z \text{ or } z = -2}$

Example 1.5 – Solving equations with the Distributive Property

When solving an equation that involves variables and numbers inside a parenthesis, it is important to apply the Distributive Property to each variable and number inside the parenthesis, and then simplify on both sides of the equal sign before trying to isolate the variables.

Given $3(5x + 4) - 8 = -3x + 10$, solve for x

Solution:

According to the order of operation, the item inside of the parenthesis must be solved first. But since $5x$ and 4 can't be combined, in order to break the parenthesis, the Distributive Property must be applied by multiplying each term inside the parenthesis with 3.

$$3(5x + 4) - 8 = -3x + 10 \rightarrow 15x + 12 - 8 = -3x + 10$$

Now, we combine like variables in one side, since $15x$ is greater than $-3x$, then all x variable will be combined on the left side. To do so, we add $3x$ in both sides.

$$15x + 12 - 8 + 3x = -3x + 10 + 3x$$

$$18x + 12 - 8 = 10 \rightarrow 18x + 4 = 10$$

To simplify, all numbers must be on the right so. For it, we subtract 4 in both sides.

$$18x + 4 - 4 = 10 - 4$$

$$18x = 6$$

To solve for x, we divide both side by 18

$$\frac{18x}{18} = \frac{6}{18} \rightarrow \boxed{x = \frac{6}{18} \text{ or } x = \frac{1}{3}}$$

Example 1.6 – Solving equations with the rational numbers

To solve an equation with a variable on one or both sides that involves fractions, first get rid of the fractions and solve the unknown variables using the methods learned in Example 1.3, 1.4, and 1.5.

Given $\frac{3}{4}m + 2 = \frac{2}{3}m + 5$, solve for m

Solution:
Multiple both sides of the equation by the Least Common Multiplier, LCM, of 4 and 3, which is 12

$\mathbf{12}\left(\frac{3}{4}m+2\right)=\mathbf{12}\left(\frac{2}{3}m+5\right)$ ➔ Apply the Distributive Property

$\left(\frac{\mathbf{12}\times 3}{4}m+\mathbf{12}\times 2\right)=\left(\frac{\mathbf{12}\times 2}{3}m+\mathbf{12}\times 5\right)$ ➔ Simplify the equation

$9m+24-\mathbf{8m}=8m+60-\mathbf{8m}$ ➔ Solve for ***m***

$m+24-\mathbf{24}=60-\mathbf{24}$ ➔ $m=36$ ➔ $\boxed{\boldsymbol{m}=\mathbf{36}}$

1.6. Equation in Engineering Technology with unknown variables

It is very important to know how to solve for unknown variables. There are scenarios where they might need to formulate an equation to analyze an object behavior, or simply a calculation where they have to estimate a constant by using given formulas.

Example 1.7 – Find the unknown value

Given the voltage formula

$$V=\frac{W}{Q}$$

Where ***V*** is the voltage between two points, in volts, ***W*** is the amount of energy, in Joules, needed to move a negative charge ***Q***, in Coulombs, from one point to the other point.

Find the energy expended moving a charge of 48.5 μC between two points if the voltage between the points is 5.2 V.

Solution: For this problem, it is important to identify the unknown variable first from the given equation. Since the voltage and the charge is given, the unknown variable here is *work*, ***W***.

$$V=\frac{\boldsymbol{W}}{Q}$$

Multiple both side of the equation with Q

$V \times Q = \frac{W}{Q} \times Q \quad \Rightarrow \quad V \times Q = W \quad \Rightarrow \quad W = V \times Q$

Substitute the given value for Q and V

$W = 48.5\ \mu C \times 5.2\ V$

$W = 48.5 \times 10^{-6} C \times 5.2\ V$

$W = 252.2 \times 10^{-6} J \Rightarrow W = 252.2\ \mu J$