Chapter 5

Parallel Circuit

Chapter 5 presents the voltage and current behavior through elements connected in parallel. It introduces the definition of parallel connectivity, current flows and voltage drops through elements connected in parallel, and the equivalent resistance of a parallel resistivity circuit.

By the end of chapter 5, students should be able to:

- identify a parallel circuit and the current and voltage through each element in the parallel circuit.
- calculate the equivalent or total resistance of a parallel resistivity circuit.
- apply Kirchhoff's Current Law, KCL, and Current Divider Rule, CDR, to find an unknown current in a parallel circuit.
- find the equivalent current source all current sources connected in parallel.

5.1. Parallel Circuit Characteristics

5.1.1. Parallel Circuit Connectivity

Elements are connected in parallel if one terminal of the elements are connected to the same node, and the other terminal of the elements are also connected to another same node.

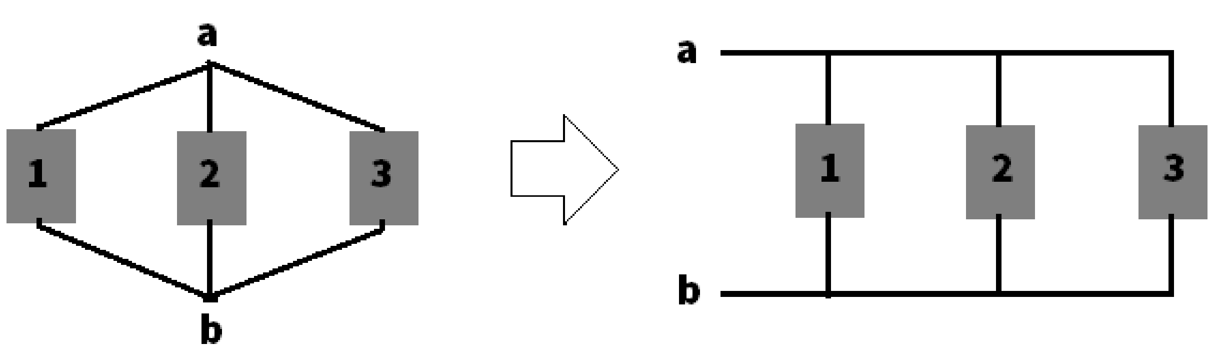


Figure 5.1. Three elements connected in parallel between node ***a*** *and* ***b***

5.1.2. Voltage behavior in elements connected in parallel

We can apply KVL to a closed loop of a parallel circuit to understand the voltage behavior across the elements in parallel. For example, taking Figure 5.1, if we apply KVL to each of the circuit closed loop,

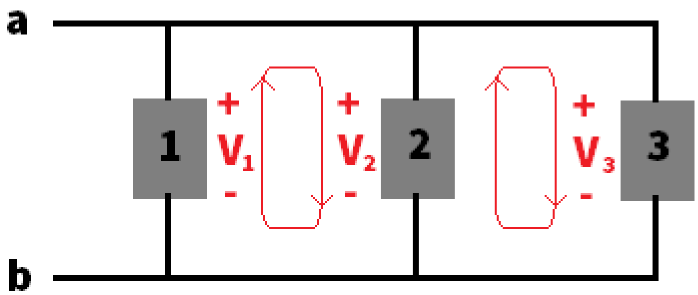


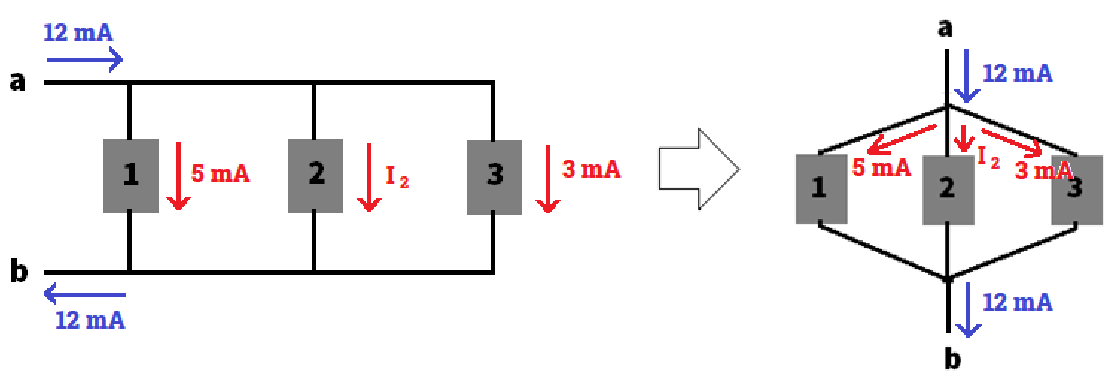
Figure 5.2 – Applying KVL to each closed loop in Figure 5.1

from the left loop we can see that $+ V_1 – V_2 = 0V$, then $V_1 = V_2$. The same way from the right loop, we can calculate $+ V_2 – V_3 = 0$ V, then $V_2 = V_3$. From Figure 5.2, we can conclude that $V_1 = V_2 = V_3$. Therefore, the voltage across each element in parallel connectivity is the same.

5.1.3. Current flowing through elements connected in parallel

As we stated, from KVL analysis in Figure 5.2, that the voltage across each element in parallel connectivity is the same, then the current flowing through each element in parallel depends on the resistivity of the element. Since according to Ohm's law, current is inversely proportional to resistance, then the greater the resistance, the lower is the current, and vice-versa. Also, according to KCL, the source or total current in a system is sum of all individual current.

Example 5.1) For the following parallel network:



We can apply KCL to find the unknown current I_2 :

Sum of all current flowing *in* to node ***a*** = Sum of all current flowing *out* to node ***a***

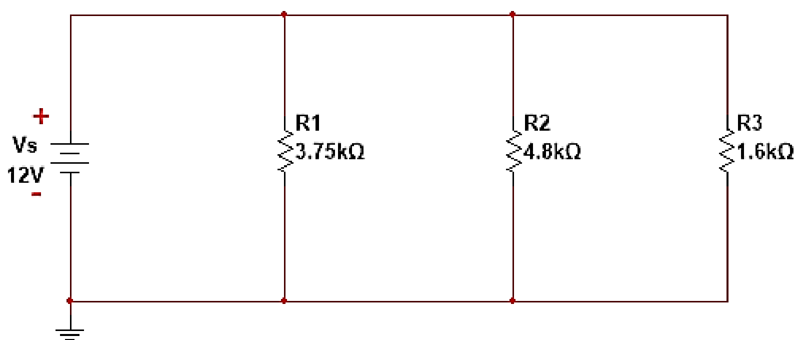
12 mA = 5 mA + I_2 + 3 mA

12 mA = 8 mA + I_2

12 mA – 8 mA = I_2

4 mA = I_2

Example 5.2) For the following parallel circuit,



We can apply Ohm's to find the current flowing to each resistor of the parallel circuit.

Firstly, we already know that the voltage across each elements connected in parallel is the same:

$V_S = V_{R1} = V_{R2} = V_{R3} = 12\ V$

Since each resistance is given and we already know the voltage across each resistor, we apply Ohm's law to find the individual current:

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{12\ V}{3.75\ k\Omega} = \frac{12\ V}{3.75\ \times 10^3\Omega} = 3.2 \times 10^{-3}A = 3.2\ mA$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{12\ V}{4.8\ k\Omega} = \frac{12\ V}{4.8\ \times 10^3\Omega} = 2.5 \times 10^{-3}A = 2.5\ mA$$

$$I_{R3} = \frac{V_{R3}}{R_3} = \frac{12\ V}{1.6\ k\Omega} = \frac{12\ V}{1.6\ \times 10^3\Omega} = 7.5 \times 10^{-3}A = 7.5\ mA$$

Once we have all the individual current, we can apply KCL to find the source or total current:

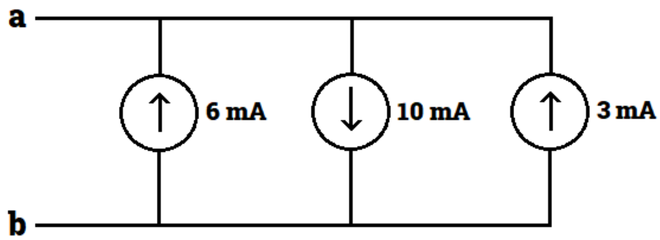
$I_S = I_{R1} + I_{R2} + I_{R3} = 3.2\ mA + 2.5\ mA + 7.5\ mA$

$I_S = 13.2\ mA$

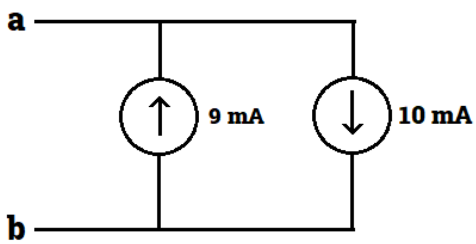
5.1.4. Equivalent current source connected in parallel

The equivalent current source of two or more current sources in parallel is equal to the algebraic sum of the individual current source. For the algebraic sum, remember that the currents add if they are flowing to the same direction, otherwise, the current subtracts if they flow opposite direction.

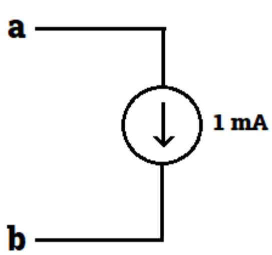
Example 5.3) For the following current sources connected in parallel,



to find the equivalent current source, we can add 6 mA and 3 mA first because both of them flow up,



After we have the sum, we find one equivalent current source by finding the difference between 9 mA and 10 mA because they flow opposite direction. Remember to keep the direction of the greatest current:



5.2. Parallel Resistivity Circuit

5.2.1. Equivalent Resistivity in a Parallel Network

The equivalent or total resistance of two or more resistors connected in parallel is the reciprocal of the total conductance:

$$Total\ Conductance = G_T = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

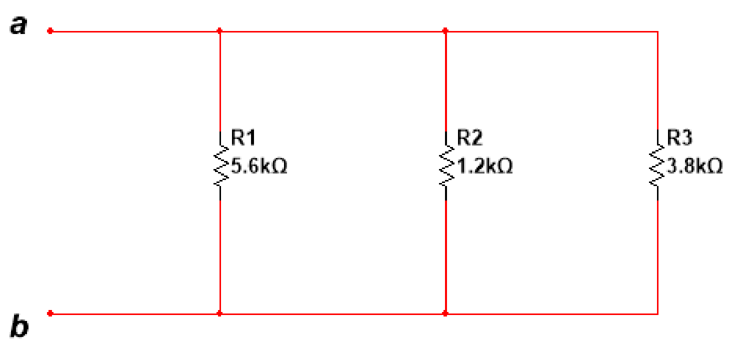
$$R_{T(parallel\ resistors)} = \frac{1}{G_T}$$

$$\boxed{R_{T(parallel\ resistors)} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}}$$

Formula 5.1 – Total resistance for elements connected in parallel

The equivalent resistance of resistors in parallel is always smaller than the smallest resistor in parallel.

Example 5.4) For the following parallel circuit, find the equivalent resistance between node ***a*** and ***b***:



We can use Formula 5.1 to find the total resistance:

$$R_{T(parallel\ resistors)} = \frac{1}{\frac{1}{5.6\ k\Omega} + \frac{1}{1.2\ k\Omega} + \frac{1}{3.8\ k\Omega}} = 0.78427\ \times 10^3\Omega = 784.27\ \Omega$$

$$R_{T(parallel\ resistors)} = 784.27\ \Omega$$

The total resistance is 784.27 **Ω**, which is smaller than the smallest resistance 1.2 k**Ω**

There is also two special cases to find the equivalent resistance:

Case 1: Product over the Sum

Case 1 applies **ONLY to two resistors connected in parallel**. For example, if we apply Formula 5.1 to find the equivalent resistance R_1 and R_2 in parallel, we will have:

$$R_{T(ONLY\ two\ resistors\ in\ parallel)} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Solving the formula:

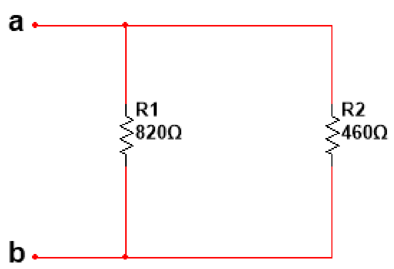
$$R_{T(ONLY\ two\ resistors\ in\ parallel)} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} = \frac{R_1 R_2}{R_2 + R_1}$$

$$R_{T(ONLY\ two\ resistors\ in\ parallel)} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\boxed{R_{T(ONLY\ two\ resistors\ in\ parallel)} = \frac{R_1 R_2}{R_1 + R_2}}$$

Formula 5.2 – Product over the Sum: ONLY applies to two resistors connected in parallel

Example 5.5) for the following circuit, find the equivalent resistance between node ***a*** and ***b***:



Applying Formula 5.1 to find the equivalent resistance:

$$R_{T(parallel\ resistors)} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{820\ \Omega} + \frac{1}{460\ \Omega}} = 294.6875\ \Omega = 294.69\ \Omega$$

Or we can use **Case 1**, Formula 5.2, since the circuit has ONLY two resistors in parallel connection:

$$R_{T(ONLY\ two\ resistors\ in\ parallel)} = \frac{R_1 R_2}{R_1 + R_2} = \frac{820\ \Omega\ \times 460\ \Omega}{820\ \Omega + 460\ \Omega} = 294.6875\ \Omega = 294.69\ \Omega$$

Case 2: Equal resistance

Case 2 applies **ONLY to all resistors connected in parallel with the same resistance**. In this case, the equivalent resistance is found by taking the resistance value divided by the number of resistors with the same resistance. For example, if we use Formula 5.1 to find the equivalent resistance of three resistors, with resistance ***R***, connected in parallel:

$$R_{T(equal\,resistance)} = \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} = \frac{1}{\frac{3}{R}} = \frac{R}{3}$$

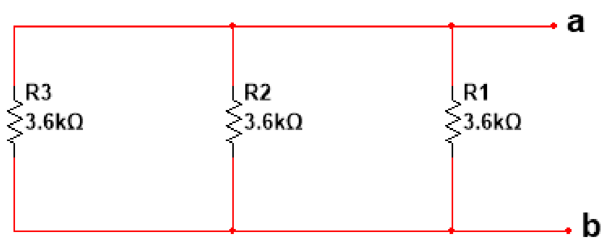
Therefore, we can write the formula as:

$$R_{T(equal\,resistance)} = \frac{R}{N}$$

Where N is the number of resistors with the resistance connected in parallel

Formula 5.3 – Equal resistance connected in parallel

Example 5.6) for the following circuit, find the equivalent resistance between node ***a*** and ***b***:



Applying Formula 5.1 to find the equivalent resistance:

$$R_{T(parallel\,resistors)} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{3.6\,k\Omega}+\frac{1}{3.6\,k\Omega}+\frac{1}{3.6\,k\Omega}} = 1.2\,k\Omega$$

Or we can use **Case 2**, Formula 5.3, since all three resistors connected in parallel has the same resistance:

$$R_{T(equal\,resistance)} = \frac{R}{N} = \frac{3.6\,k\Omega}{3} = 1.2\,k\Omega$$

5.2.2. Power distribution in a parallel resistivity circuit

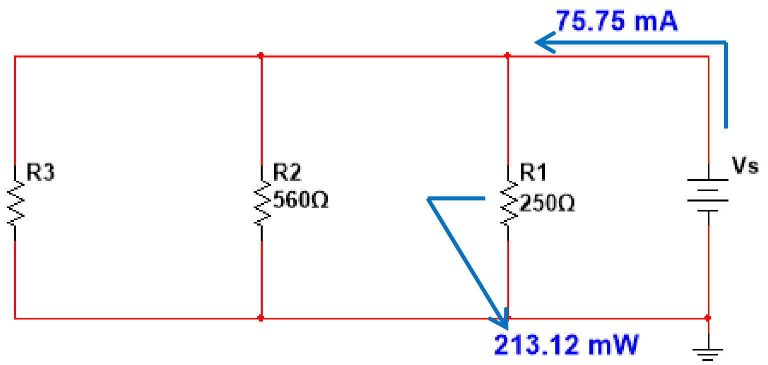
The total power, P_T, or source power, P_S, consumed in a parallel circuit is equal to the sum of the power consumed by the individual resistors.

$$P_S = P_{R1} + P_{R2} + P_{R3} + \cdots\ P_{R_N}$$

$$\textit{where } R_N \textit{ means up to number of resistors connected in parallel}$$

Formula 5.4 – Total power calculation in a parallel circuit

Example 5.7) for the following circuit, apply the parallel circuit concepts to find the unknown values, I_{R1}, I_{R2}, I_{R3}, R_3, V_S, and P_{R2}:



When we look at the circuit, we check if there is any elements that provide two information, in this case, R_1 and P_{R1} are given, then, we can apply the Power's Law to find V_{R1} or I_{R1}. Since all four elements are connected in parallel, we can find V_{R1}, which is going to be the same voltage for all four elements:

$$P_{R1} = \frac{{V_{R1}}^2}{R_1}$$

$$213.12\ mW = \frac{{V_{R1}}^2}{250\ \Omega}$$

$$213.12\ \times 10^{-3} W \times 250\ \Omega = {V_{R1}}^2$$

$$\sqrt{53.28} = \sqrt{{V_{R1}}^2} \quad \rightarrow \quad 7.3\ V = V_{R1}$$

$$V_{R1} = V_{R2} = V_{R2} = V_S = 7.3\ V$$

$$V_S = 7.3\ V$$

Once we have the voltage, we can find the missing current, IR1 and IR2, using Ohm's law

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{7.3\ V}{250\ \Omega} = 0.0292\ A = \ 29.2\ mA$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{7.3\ V}{560\ \Omega} = 0.013038\ A = \ 13.04\ mA$$

To find the third current, I_{R3}, we can apply KCL:

$$I_{R1} + I_{R2} + I_{R3} = I_S$$

$$29.2\ mA + 13.04\ mA + I_{R3} = 75.75\ mA$$

$$42.24\ mA + I_{R3} = 75.75\ mA$$

$$I_{R3} = 33.51\ mA$$

Since we already have the current and voltage across R_3, we can apply Ohm's law to find R_3:

$$R_3 = \frac{V_{R3}}{I_{R3}} = \frac{7.3\ V}{33.51\ mA} = \frac{7.3\ V}{33.51\ \times 10^{-3} A} = 217.85\ \Omega$$

Lastly, we use I_{R2} and V_{R2} to find the power dissipation in R_2

$$P_{R2} = V_{R2} \times I_{R2} = 7.3\ V \times 13.04\ mA = 7.3\ V \times 13.04\ \times 10^{-3} A = 95.19 \times 10^{-3} W = 95.19\ mA$$

References

Johnson, D. (2003). Origins of the equivalent circuit concept: the voltage-source equivalent. *Proceedings of the IEEE*. *91*, 636-640.