Chapter 4

Series Circuit

Electrical components/elements can be connected to form a network or circuit. Depending on the elements interconnection we can determine the current and voltage distribution among the elements and determine whether they are series, parallel, or series-parallel circuit.

Chapter 4 presents the voltage and current behavior through elements connected in series. It introduces the definition of series circuit, current flows and voltage drops through elements connected in series, and the equivalent resistance of a series resistivity circuit.

By the end of chapter 4, students should be able to:

- identify a series circuit and the current and voltage through each element in the series circuit.
- calculate the equivalent or total resistance of a series resistivity circuit.
- apply Kirchhoff's Voltage Law, KVL, and Voltage Divider Rule, VDR, to find an unknown voltage in a series circuit.
- find the equivalent voltage of a series voltage sources.

4.1. Series Circuit Characteristics

4.1.1. Series Circuit Connectivity

To understand series circuit, let's see the connection of two elements in series first. Two elements are connected in series if one of their terminals are connected to the same node.

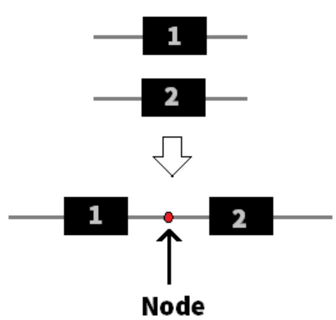


Figure 4.1. Two elements connected in series

Then, elements are connected in series if they are arranged in a chain with only two elements connected in each node.

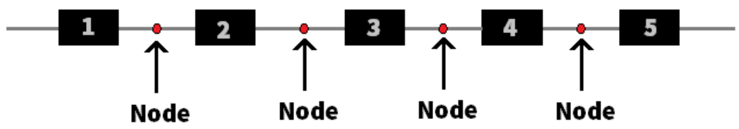


Figure 4.2. Elements connected in series

4.1.2. Current flowing through elements in a series circuit

If we apply the Kirchhoff's Current Law, KCL, to analyze the current through elements connected in series, we can find that current leaving one element is the same coming into the next element in series.

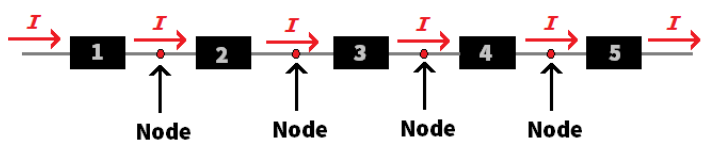
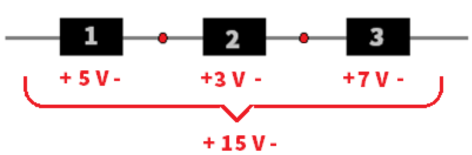


Figure 4.3. Current flowing in a series circuit

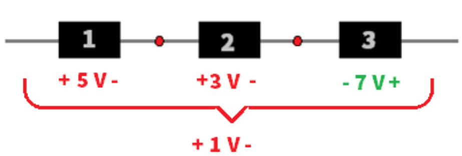
4.1.3. Voltage distribution in elements connected in series

The total voltage applied to a series network is equal to the algebraic sum of the individual voltage drops/applies at each of the elements connected in series.

Example 4.1) For the following series network:

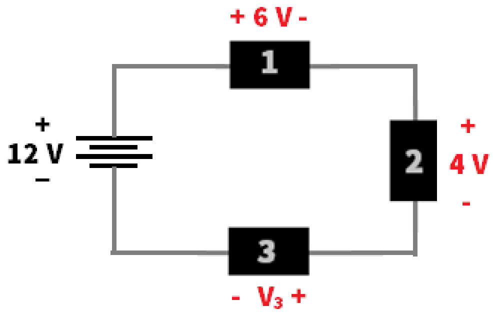


if we apply Kirchhoff's Voltage Law to the network, the total voltage across the three elements in series is +5 V +3 V +7 V = +15 V. On the other hand, if we swap the polarity of the third element,



then when KVL is applied to the network, the total voltage across the three elements in series is +5 V +3 V -7 V = +1 V, and it keeps the polarity of the greatest voltage.

Example 4.2) For the following series circuit,



We can find the missing voltage, V_3, if we apply KVL to the closed circuit as,

$+ 12\ V – 6\ V – 4\ V – V_3 = 0\ V$

After KVL, we can apply algebraic math to solve for the unknown voltage, V_3:

$+ 2\ V – V_3 = 0\ V$

$\mathbf{+ 2\ V = V_3}$

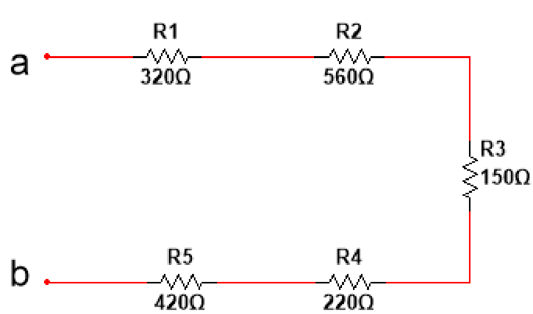
4.2. Series Resistivity Circuit

4.2.1. Equivalent Resistivity in a Series Network

In electronics, an equivalent circuit refers to a theoretical circuit that retains all of the electrical characteristics of a given circuit. Often, an equivalent circuit is sought that simplifies calculations, and more broadly, that is a simplest form of a more complex circuit in order to aid analysis. *(Johnson, 2003)*

The equivalent resistance of two or more resistors connected in series is the sum of the individual resistance.

Example 4.3) For the following network of five resistors,



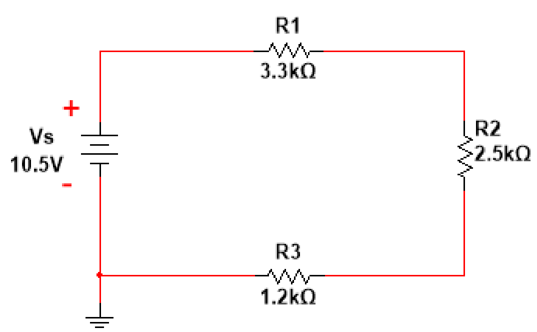
the equivalent or total resistance between node **a** and **b** is 320Ω + 560Ω + 150Ω + 220Ω + 420Ω = 1670Ω = 1.67 kΩ

4.2.2. Voltage distribution in a series resistivity circuit

Since the current flowing through each resistor in a series circuit is the same, the voltage drop across a resistor in a series circuit is directly proportional to the size of the resistor.

Example 4.4) For the following series circuit:

a. Find the total resistance, R_T
b. The total or source current, I_S, and the current through each resistor I_{R1}, I_{R2}, and I_{R3}
c. The voltage drop at each resistor, V_{R1}, V_{R2}, and V_{R3}



Answer:

a. Since R_1, R_2, and R_3 are connected in series, then the total resistance will be the sum of them:
   $3.3\ k\Omega + 2.5\ k\Omega + 1.2\ k\Omega = 7\ k\Omega$
   $R_T = 7\ k\Omega$

b. To find the total or source current, we can use Ohm's law:

$$I_S = \frac{V_S}{R_T} = \frac{10.5\ V}{7\ k\Omega} = \frac{10.5\ V}{7\ \times 10^3 \Omega} = 1.5 \times 10^{-3} A = 1.5\ mA$$

$$I_S = 1.5\ mA$$

   If we apply KCL to the series circuit, we can confirm that only one current is flowing to each element in the series circuit, therefore, the current to each element is same as the source current.

$$I_S = I_{R1} = I_{R2} = I_{R3} = 1.5\ mA$$

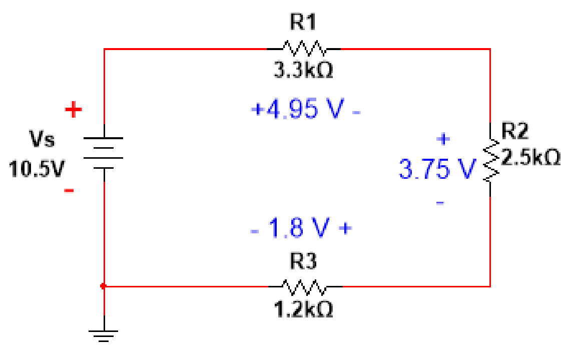
c. Once we know the current flowing within the series circuit, we can apply Ohm's law to find the voltage across each resistor:

$$V_{R1} = I_{R1} \times R_1 = (1.5\ mA) \times (3.3\ k\Omega) = (1.5\ \times 10^{-3} A) \times (3.3\ \times 10^{3} \Omega) = 4.95\ V$$

$$V_{R2} = I_{R2} \times R_2 = (1.5\ mA) \times (2.5\ k\Omega) = (1.5\ \times 10^{-3} A) \times (2.5\ \times 10^{3} \Omega) = 3.75\ V$$

$$V_{R3} = I_{R3} \times R_3 = (1.5\ mA) \times (1.2\ k\Omega) = (1.5\ \times 10^{-3} A) \times (1.2\ \times 10^{3} \Omega) = 1.8\ V$$

We can also check if our calculation is correct by applying KVL with a conventional current flowing clockwise:



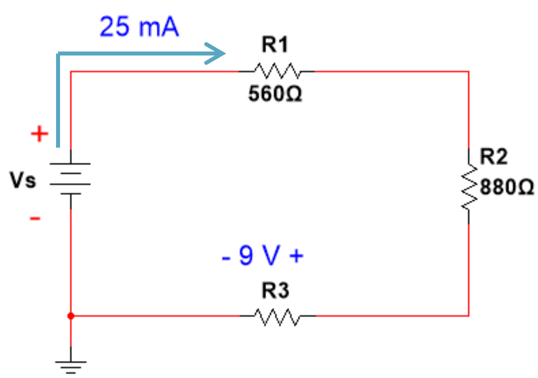
Sum of all voltages = 0 V

+10.5 V – 4.95 V – 3.75 V – 1.8 V = 0V

0 V = 0 V ✓

Example 4.5) For the following series circuit:

a. Find the resistance of R_3
b. Find the source voltage, V_S
c. Find the total resistance, R_T



Answer:

a. Since there is only current flowing through each element in a series circuit, then:

$$I_S = I_{R1} = I_{R2} = I_{R3} = 25\ mA$$

Also, as V_{R3} is given as 9 V, then we just need to apply Ohm's law to find the resistance in R_3:

$$R_3 = \frac{V_{R3}}{I_{R3}} = \frac{9\ V}{25\ mA} = \frac{9\ V}{25\ \times 10^{-3} A} = 360\ \Omega$$

$$R_3 = 360\ \Omega$$

b. To find the source voltage, V_S, we can find the voltage across R_1 and R_2 first, and then apply KVL to find the total or source voltage:

$$V_{R1} = I_{R1} \times R_1 = 25\ mA \times 560\ \Omega = \ 25\ \times 10^{-3} A \times 560\ \Omega = 14\ V$$

$$V_{R2} = I_{R2} \times R_2 = 25\ mA \times 880\ \Omega = \ 25\ \times 10^{-3} A \times 880\ \Omega = 22\ V$$

KVL:

$$V_S - \ V_{R1} - \ V_{R2} - V_{R3} = 0V$$

$$V_S - \ 14\ V - \ 22\ V - 9\ V = 0V$$

$$V_S - \ 45\ V = 0V$$

$$V_S = 45\ V$$

Voltage Divider Rule, VDL

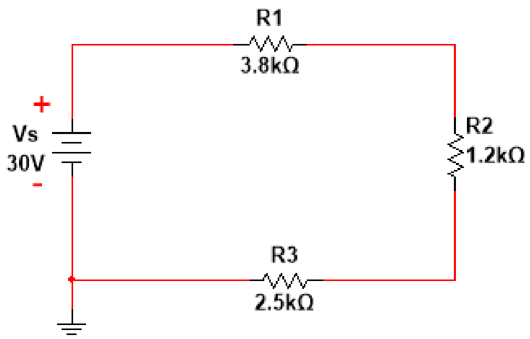
In electronics, a voltage divider (also known as a potential divider) states that the voltage across a resistor is directly proportional to the ratio of the resistance with respect to the total or equivalent resistance in series connection:

$$V_{R_x} = \left(\frac{R_x}{R_{T(series\ connection)}}\right) \times V_S$$

Formula 4.1 – Voltage Divider Rule to find the voltage across R_x

Therefore, the higher the resistance the higher will be voltage drop.

Example 4.6) For the following circuit, use Voltage Divider Rule to find the voltage across each resistor:



Answer:

To apply the Voltage Divider Rule, we need to find the total resistance of resistor connected in series first:

$$R_T = R_1 + R_2 + R_3 = 3.8\ k\Omega + 1.2\ k\Omega + 2.5\ k\Omega = 7.5\ \text{k}\Omega$$

Once we have the total resistance, now we can calculate the voltage across each resistor in the series circuit:

$$V_{R1} = \left(\frac{R_1}{R_{T(series\ connection)}}\right) \times V_S = \left(\frac{3.8\ k\Omega}{7.5\ k\Omega}\right) \times 30\ V = 15.2\ V$$

$$V_{R2} = \left(\frac{R_2}{R_{T(series\ connection)}}\right) \times V_S = \left(\frac{1.2\ k\Omega}{7.5\ k\Omega}\right) \times 30\ V = 4.8\ V$$

$$V_{R3} = \left(\frac{R_3}{R_{T(series\ connection)}}\right) \times V_S = \left(\frac{2.5\ k\Omega}{7.5\ k\Omega}\right) \times 30\ V = 10\ V$$

We can apply KVL to check our calculation:

30 V – 15.2 V – 4.8 V – 10 V = 0 V

30 V – 30 V = 0 V

0 V = 0 V ✓

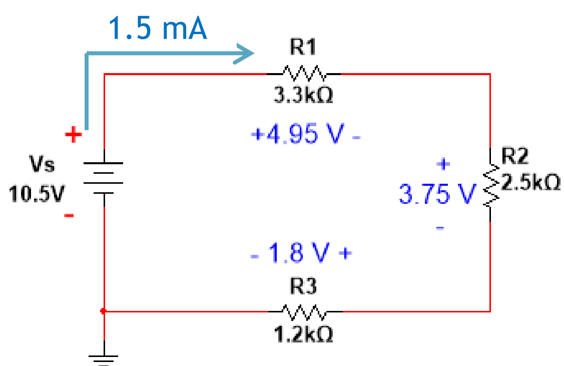
4.2.3. Power distribution in a series resistivity circuit

As the current flowing through each resistor in a series circuit is the same but the voltage drop in each resistor is different, depending on the resistance, then the power dissipation by each individual resistor in a series circuit is also different, and their combined total adds up to the power source, P_S, or total power, P_T:

$$P_S = P_{R1} + P_{R2} + P_{R3} + \cdots\ P_{R_N}$$

$$where\ R_N\ means\ up\ to\ number\ of\ resistors\ connected\ in\ series$$

Example 4.7) For the circuit in Example 4.4, find the power dissipated by each resistor and by the voltage source:



Since we know at least two information of each resistor, we can apply the power formula to find the power dissipated in each resistor:

$P_{R1} = \frac{V_{R1}{}^2}{R_1}$ or $P_{R1} = V_{R1} \times I_{R1}$ or $P_{R1} = I_{R1}{}^2 \times R_1$

Let's use $P_{R1} = V_{R1} \times I_{R1}$

$P_{R1} = 4.95\ V \times 1.5\ mA = 4.95\ V \times 1.5\ \times 10^{-3} A = 7.425\ \times 10^{-3} W = 7.425\ mW = 7.43\ mW$

$P_{R2} = 3.75\ V \times 1.5\ mA = 3.75\ V \times 1.5\ \times 10^{-3} A = 5.625\ \times 10^{-3} W = 5.625\ mW = 5.63\ mW$

$P_{R3} = 1.8\ V \times 1.5\ mA = 1.8\ V \times 1.5\ \times 10^{-3} A = 2.7\ \times 10^{-3} W = 2.7\ mW$

$P_S = 10.5\ V \times 1.5\ mA = 10.5\ V \times 1.5\ \times 10^{-3} A = 15.75\ \times 10^{-3} W = 15.75\ mW$

We can also add the individual power dissipation to find the total power:

$P_S = P_{R1} + P_{R2} + P_{R3} = 7.425\ mW + 5.625\ mW + 2.7\ W = 15.75\ mW$

References

Johnson, D. (2003). Origins of the equivalent circuit concept: the voltage-source equivalent. *Proceedings of the IEEE*. *91*, 636–640.